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TOTAL FLUX DENSITY VARIATIONS IN EXTRAGALACTIC RADIO SOURCES. III. DOPPLER BOOSTING FACTORS, LORENTZ FACTORS, AND VIEWING ANGLES FOR ACTIVE GALACTIC NUCLEI

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ABSTRACT

We have calculated Doppler boosting factors, $D_{\rm var}$, for a sample of active galactic nuclei (AGNs) using total flux density variation monitoring data at 22 and 37 GHz. We argue that this method is more accurate than the other commonly used methods based on the synchrotron self-Compton X-ray flux or equipartition of energy. We compare our Doppler factors with other results and conclude that even if the average $D_{\rm var}$ for a class of sources is very similar to all others, the variability Doppler factors for individual objects are more accurate and reliable. An important application of precise Doppler factors is presented, namely, calculating the Lorentz factors, Γ, and the viewing angles, θ, of relativistic outflows in AGNs. We find that high-polarization quasars have the greatest Doppler boosting, while low-polarization quasars and BL Lac objects are less boosted. The two groups of quasars show different characteristics because of different combinations of the Lorentz factor and viewing angle, rather than either a different Γ or θ alone.

Subject headings: galaxies: active — quasars: general — radiation mechanisms: nonthermal

1. INTRODUCTION

A reliable determination of the Doppler boosting factor, D, is a key step in studying the origins of the physical processes in the compact emission regions of active galactic nuclei (AGNs). A typical AGN, in the simplest case, can be modeled by a central black hole, an accretion disk around it, and two relativistic jets emanating from the core. The relativistic outflow in an AGN of this kind can be defined by just two intrinsic parameters. These are the speed of the jet flow, or the Lorentz factor Γ, and the viewing angle θ (the angle between the line of sight and the outflow axis). These can be calculated if the apparent superluminal speed $\beta_{\rm app}$ and the Doppler factor of the jet are measured. It is possible to derive $\beta_{\rm app}$ directly from VLBI monitoring. D, however, can be determined in various ways, with varying accuracy.

The usual way of estimating D is from synchrotron self-Compton X-ray flux, by comparing the observed and predicted fluxes. The observed X-ray flux is assumed to be caused by inverse Compton scattering of synchrotron photons off the radiating particles. The prediction of the amount of X-rays is made on the basis of VLBI data. An excess of predicted X-rays as compared to the actual observed X-rays is taken as a sign of Doppler boosting, relativistically enhancing the observed radio flux. The relation of the two X-ray values gives the synchrotron self-Compton Doppler factor, $D_{\rm SSC}$. The most severe limitation of the SSC-method is the assumption that the observed VLBI frequencies, fluxes, and sizes correspond to the turnover values. $D_{\rm SSC}$ strongly depends on the turnover values, so any error in them is greatly enlarged. In addition, the VLBI and X-ray observations are taken at different times, sometimes even years apart, so it is usually impossible to identify the specific component where the X-ray emission originates. In addition, the VLBI properties are strongly variable in time and thus do not give reliable information on the characteristic long-term properties of the sources. The largest collection of SSC Doppler factors $D_{\rm SSC}$ (for 105 sources with available VLBI data) has been calculated by Ghisellini et al. (1993).

Another way of estimating the Doppler factor is to assume equipartition of energy between the radiating particles and the magnetic field. Readhead (1994) suggests that the maximum intrinsic brightness temperature, $T_{b,\rm int}$, for powerful synchrotron radio sources is the equipartition brightness temperature, $T_{\rm eq} = 5 \times 10^{10}$ K, rather than the usual value of 10^{12} K based on the inverse-Compton catastrophe theory. This result has been confirmed by Lähteenmäki, Valtaoja, & Wiik (1999, hereafter Paper II), who estimated a value of $\leq 10^{11}$ K for the maximum brightness temperature. An observed VLBI brightness temperature, $T_{b,\rm VLBI}$, in excess of $T_{\rm eq}$ therefore indicates Doppler boosting, with the equipartition Doppler factor defined as $D_{\rm eq} = T_{b,\rm VLBI}/T_{\rm eq}$ (here, as everywhere in this paper, the brightness temperatures are given in the source frame). This can be estimated from single-epoch radio observations, which is a considerable advantage, and it depends only weakly on the observed values. However, the values needed for the calculations should again be obtained at the turnover frequency. Güijosa & Daly (1996; hereafter G&D) calculated $D_{\rm eq}$ for the Ghisellini et al. (1993) sample and compared them with $D_{\rm SSC}$. They found that the two estimates of the Doppler factor correlate, but that this could have been due to their similar dependence on the observed VLBI quantities, namely the flux, size, and frequency, and concluded that the matter required further investigation. This was accomplished by Guerra & Daly (1997), who also further calculated the Lorentz factors and the viewing angles of outflows from AGN cores using updated values of $D_{\rm eq}$ and $D_{\rm SSC}$. The results from both G&D and Guerra &

Daly (1997) will be discussed and compared with our results in §§ 3 and 4.

In this paper we present a third way of estimating the Doppler factors, which we argue to be more accurate and reliable than the above-mentioned methods. Ours is an improvement of a previous study made by Teräsranta & Valtaoja (1994). We have used new data, a larger sample, and a more accurate way of estimating the variability parameters. We estimate the Doppler factors from total flux density (TFD) flares associated with new VLBI components emerging from the AGN core (Valtaoja et al. 1999, hereafter Paper I). The TFD monitoring data used here cover nearly 20 yr of variations, and an accurate estimate of $D_{\rm var}$ is easily obtained from careful model fits by determining the associated variability brightness temperature, $T_{b,\rm var}$. A comparison of $T_{b,\rm var}$ with $T_{\rm eq}$ gives the amount of boosting in a source. The idea is basically identical to that of Readhead (1994), but instead of $T_{b,\rm VLBI}$ we use $T_{b,\rm var}$. The dependence of $D_{\rm var}$ on the observed brightness temperature is much weaker than for $D_{\rm SSC}$ or $D_{\rm eq}$ (third root as compared to first power). TFD data at a single frequency are also easily gathered, in contrast to simultaneous VLBI and X-ray data. VLBI observations give only single-epoch information on the brightness temperature, while $T_{b,\rm var}$, being practically constant during a high-frequency flare as well as from one outburst to another, gives a better controlled estimate of the characteristic brightness temperatures of the new components.

During the maximum phase of a shock component's development, typically occurring between 10 and 100 GHz (e.g., Valtaoja et al. 1988), all strong shocks seem to reach the maximum, $T_{b,\rm int} = T_{\rm eq}$ (Paper II), and can therefore be used to estimate $D_{\rm eq}$ or $D_{\rm var}$, but only during this time and at these frequencies. If a long time, typically 1 yr or more, has elapsed since the latest shock event, the ejected VLBI component is already an old, adiabatically expanding remnant with a brightness temperature below the maximum possible $T_{b,\rm int}$ and can no longer be used to estimate $D_{\rm eq}$. The total flux density has also returned to close to the quiescent level, with an estimated $T_{b,\rm var}$ smaller than the maximum possible $T_{b,\rm int}$. Similarly, most VLBI data are taken at centimeter wavelengths, and relate to the shock remnants that have already dropped well below the maximum $T_{b,\rm int}$. Observations at high radio frequencies ($\geq$22 GHz) are essential for estimating both $D_{\rm eq}$ and $D_{\rm var}$.

$D_{\rm var}$ is also a totally independent estimate of Doppler boosting in a source, and can be used to test the reliability of the other methods. The variability Doppler factor will be further used in the calculation of the intrinsic Lorentz factor and the viewing angle of the outflow for each source in order to demonstrate the power of reliable and accurate D.

2. OUR SAMPLE

We have used the multifrequency continuum monitoring data from the Metsähovi Radio Observatory to model the flux variations. The data have been gathered during almost 20 yr of continuum monitoring of extragalactic radio sources at 22 and 37 GHz, totalling over 30,000 TFD measurements of $\sim$130 objects (Teräsranta et al. 1992, 1998). We decomposed each flux curve into exponential flares, obtaining the necessary variability parameters for determining $D_{\rm var}$, in particular the associated brightness temperature of each flare, $T_{b,\rm var}$. The details of this procedure are described in Papers I and II. The observed variability brightness temperature (in the source proper frame) is

$$T_{b,\rm var} = 5.87 \times 10^{21}\, h^{-2}\, \frac{\lambda^2 S_{\rm max}}{\tau_{\rm obs}^2} (\sqrt{1+z} - 1)^2 , \quad (1)$$

where λ is the observed wavelength in meters, z is the redshift, $S_{\rm max}$ is the maximum amplitude of the outburst in janskys, and $\tau_{\rm obs} = dt/d(\ln S)$ is the observed variability timescale in days (cf. Paper I). The numerical factor in equation (1) corresponds to using $H_0 = 100\ h$ km s^{-1} Mpc^{-1} and $q_0 = 0.5$ and to assuming that the source is a homogeneous sphere. The variability Doppler factors could be estimated for a total of 81 sources using the familiar equation

$$D_{\rm var} = \left(\frac{T_{b,\rm var}}{T_{b,\rm int}}\right)^{1/3} . \quad (2)$$

For $T_{b,\rm int}$, we used the equipartition value of 5×10^{10} K suggested by Readhead (1994). (In Paper II we demonstrated that during flares, $T_{b,\rm int} \simeq T_{\rm eq}$ independent of the amount of Doppler boosting.) Twenty-seven of these sources were high-polarization quasars (HPQs, optical polarization $>3\%$), 26 were low-polarization quasars (LPQs, optical polarization $<3\%$), 20 were BL Lac objects (BLOs), and eight were radio galaxies (GALs). There are 48 sources in common between our sample and the G&D $D_{\rm SSC}$ and $D_{\rm eq}$ sample. We then calculated the intrinsic Lorentz factors and viewing angles for 45 sources with available VLBI expansion speeds. The Lorentz factor is given by

$$\Gamma_{\rm var} = \frac{\beta_{\rm app}^2 + D_{\rm var}^2 + 1}{2D_{\rm var}} \quad (3)$$

and the viewing angle by

$$\theta_{\rm var} = \arctan \frac{2\beta_{\rm app}}{\beta_{\rm app}^2 + D_{\rm var}^2 - 1} . \quad (4)$$

In this set of sources, there are 19 HPQs, 12 LPQs, nine BLOs, and five GALs. There are 19 sources in common between our sample and the Guerra & Daly (1997) sample of SSC and equipartition Lorentz factors and viewing angles. Our basic sample includes $\approx$80 brightest flat-spectrum northern and equatorial AGNs, and it also includes the complete flux-limited 2 Jy sample defined by Valtaoja, Lähteenmäki, & Teräsranta (1992), with the exception of 0716+714.

3. THE VARIABILITY DOPPLER FACTORS

3.1. *The Variability Doppler Factor Distributions*

The Doppler factors $D_{\rm var}$ calculated from TFD variations are presented in Table 1. Also given are the source classification; the redshift, z; the frequency, ν; $D_{\rm SSC}$ from G&D; the variability brightness temperature, log $T_{b,\rm var}$; the apparent superluminal speed, $\beta_{\rm app} = v/c$, when available, and its reference; the Lorentz factor, $\Gamma_{\rm var}$; and the viewing angle, $\theta_{\rm var}$. The classification of sources follows the guidelines of our previous work, e.g., Teräsranta & Valtaoja (1994). Some of the sources are borderline cases or otherwise ill-defined, and may have been classified differently by various authors. Compared to the sample of Guerra & Daly (1996), there are four out of 19 common sources with a different classification (in the G&D sample, five out of the 48 common sources). These small differences in classification are not significant to our overall conclusions. We have collected the VLBI expansion speeds mainly from Vermeulen & Cohen (1994), but we have updated these values and gathered new

TABLE 1

Doppler Boosting Factors for All Sources

Source	z	Type	ν	$\log T_{b,\mathrm{var}}$	D_{var}	D_{SSC}	β_{app}	Reference	Γ_{var}	θ_{var}
0003−06	0.347	BLO	37	11.24	1.52	...	...	...	...	...
0007+106......	0.089	GAL	37	11.84	2.41	...	...	...	...	...
0016+731......	1.781	LPQ	22	14.49	18.37	7.9	8.35	8	11.11	2.35
0106+013......	2.107	HPQ	22	13.51	8.62	15	8.2	8	8.27	6.65
0133+476......	0.86	HPQ	22	13.25	7.09	13	...	...	...	...
0149+218......	1.32	LPQ	37	12.72	4.72	...	...	...	...	...
0202+149......	0.833	HPQ	37	13.02	5.93	...	0.4	7	3.06	1.34
0212+735......	2.367	HPQ	37	12.56	4.16	7.1	3.88	8	4.01	13.89
0219+428......	0.444	BLO	22	11.60	1.99	0.077	14.89	6	56.97	7.55
0234+285......	1.207	HPQ	22	13.29	7.29	13	9.29	8	9.63	7.64
0235+164......	0.94	BLO	22	14.34	16.32	5	7.1	7	9.74	2.57
0248+430......	1.311	LPQ	22	11.34	1.64	...	...	...	...	...
0306+102......	0.863	BLO	22	13.70	10.04	1.6	...	...	...	...
0316+413......	0.017	GAL	37	9.24	0.33	1.2	0.43	8	1.97	50.54
0333+321......	1.259	LPQ	37	13.13	6.48	13	4.77	8	5.07	8.51
0336−01	0.852	HPQ	22	14.54	19.01	12	8.9	7	11.61	2.32
0415+379......	0.049	GAL	22	11.80	2.33	...	3.42	8	3.89	22.96
0420−014......	0.915	HPQ	22	13.91	11.72	13	4.8	4	6.89	3.45
0422+004......	0.31	BLO	37	11.39	1.70	...	...	...	...	...
0430+052......	0.033	GAL	22	10.67	0.98	4.1	3.53	8	7.39	29.63
0440−00	0.844	HPQ	22	13.88	11.46	...	6.1	6	7.40	4.17
0446+11	1.207	GAL	22	12.77	4.90	...	...	...	...	...
0454+039......	1.343	LPQ	22	11.60	1.99	...	...	...	...	...
0458−020......	2.286	HPQ	22	14.45	17.80	...	4.09	6	9.40	1.41
0528+134......	2.07	LPQ	37	14.16	14.22	2	5.15	4	8.08	2.59
0552+398......	2.365	LPQ	22	14.16	14.20	2.2	1.72	8	7.24	0.97
0605−08	0.872	HPQ	37	12.67	4.53	...	4.4	3	4.51	12.75
0642+449......	3.406	LPQ	37	14.38	16.91	...	...	...	...	...
0723−008......	0.127	GAL	22	11.89	2.50	1.3	...	...	...	...
0735+178......	0.424	BLO	22	12.20	3.17	5.6	5.84	8	7.12	15.15
0736+017......	0.191	HPQ	22	12.17	3.08	...	...	...	...	...
0754+100......	0.66	BLO	22	12.92	5.52	0.85	...	...	...	...
0804+499......	1.43	HPQ	22	14.95	26.21	16	...	...	...	...
0814+425......	0.258	BLO	22	13.00	5.84	...	...	...	...	...
0827+243......	2.046	LPQ	22	14.27	15.46	...	12.08	6	12.48	3.60
0836+710......	2.17	HPQ	37	13.78	10.67	6.7	7.69	8	8.15	5.11
0846+513......	1.86	BLO	37	13.12	6.40	...	...	...	...	...
0851+202......	0.306	BLO	37	14.47	18.03	6.8	2.79	8	9.26	0.96
0923+392......	0.699	LPQ	22	11.75	2.25	8.9	3.97	8	4.85	21.83
0945+40	1.252	LPQ	22	13.71	10.10	...	...	...	...	...
0953+254......	0.712	LPQ	22	12.75	4.83	...	...	...	...	...
0954+55	0.901	HPQ	22	12.69	4.63	...	...	...	...	...
0954+658......	0.367	BLO	22	13.16	6.62	3.8	5.7	5	5.84	8.61
1055+018......	0.888	HPQ	22	13.37	7.78	...	2.3	3	4.29	4.06
1156+295......	0.729	HPQ	22	13.62	9.42	4.9	5.2	7	6.20	5.17
1219+285......	0.102	BLO	37	11.28	1.56	0.15	2	5	2.38	36.41
1222+216......	0.435	LPQ	22	13.43	8.16	1	1.41	8	4.26	2.39
1226+023......	0.158	LPQ	22	12.97	5.71	4.6	6.1	8	6.20	10.06
1253−055......	0.538	HPQ	37	14.37	16.77	14	4.87	1	9.12	1.83
1308+326......	0.992	HPQ	22	13.87	11.38	5.2	10.8	8	10.86	5.03
1406−076......	1.494	LPQ	37	13.45	8.26	...	...	...	...	...
1413+135......	0.247	BLO	37	13.29	7.33	...	...	...	...	...
1418+546......	0.152	BLO	37	12.02	2.76	...	...	...	...	...
1502+106......	1.833	HPQ	22	13.84	11.13	...	...	...	...	...
1510−089......	0.361	HPQ	37	14.06	13.18	11	3.77	6	7.17	2.31
1538+149......	0.605	BLO	37	12.15	3.04	1	...	...	...	...
1606+106......	1.226	LPQ	22	13.61	9.32	...	2.9	7	5.16	3.52
1611+343......	1.401	LPQ	22	12.81	5.04	...	11.4	7	15.52	8.40
1633+382......	1.814	LPQ	22	13.54	8.83	2.2	4.8	9	5.78	5.48
1637+574......	0.745	LPQ	22	13.30	7.35	...	...	...	...	...
1641+399......	0.595	HPQ	22	13.32	7.45	4.1	6.32	3,8	6.47	7.63
1725+044......	0.2968	LPQ	22	11.87	2.46	...	...	...	...	...
1739+522......	1.379	HPQ	22	13.95	12.12	5.6	...	...	...	...
1741−03	1.054	HPQ	22	13.55	8.92	3.3	...	...	...	...

TABLE 1—*Continued*

Source	z	Type	ν	log $T_{b,\mathrm{var}}$	D_{var}	D_{SSC}	β_{app}	Reference	Γ_{var}	θ_{var}
1749+096......	0.32	BLO	22	14.30	15.85	11	...	...	...	...
1803+784......	0.684	BLO	37	13.13	6.45	6.6	1.8	5,8	3.55	4.70
1807+698......	0.051	BLO	22	11.47	1.80	0.54	...	...	...	...
1845+797......	0.0546	GAL	22	10.89	1.16	0.38	1.82	2	2.44	44.83
1928+738......	0.303	LPQ	22	12.41	3.71	3.4	4.97	8	5.32	14.85
2005+40.......	1.736	LPQ	37	13.51	8.63	...	...	...	...	...
2007+776......	0.342	BLO	22	12.83	5.13	3.6	2.33	8	3.19	8.62
2021+614......	0.227	GAL	37	11.30	1.59	1.1	0.2	8	1.12	14.42
2134+004......	1.936	LPQ	22	13.88	11.49	27	...	...	...	...
2144+092......	1.113	LPQ	22	13.02	5.96	...	...	...	...	...
2145+067......	0.99	LPQ	37	13.38	7.81	21	...	...	...	...
2200+420......	0.07	BLO	22	12.47	3.91	3.4	3.28	5,8	3.46	14.68
2201+315......	0.298	LPQ	22	11.60	2.00	...	...	...	...	...
2223−052......	1.404	HPQ	37	13.87	11.38	16	0	8	5.73	0
2227−08.......	1.562	HPQ	22	13.98	12.42	...	...	...	...	...
2230+114......	1.037	HPQ	22	14.16	14.23	1.5	8.86	9	9.91	3.62
2251+158......	0.859	HPQ	37	14.72	21.84	4.6	7.19	6	12.12	1.56

REFERENCES.—(1) Abraham, Carrara, & Zensus 1998; (2) Alef et al. 1996; (3) Bondi et al. 1996; (4) Britzen et al. 1998; (5) Gabuzda et al. 1994; (6) Marchenko et al. 1998; (7) Piner 1998; (8) Vermeulen & Cohen 1994; (9) Xu, Wehrle, & Marscher 1998.

ones from the latest relevant papers and proceedings. The list of references is given in Table 1. In cases where several reliable values are given for one source (multiple ejections), we have taken the mean value for β_{app}. We also prefer values obtained at high frequencies, matching our variability data, for the reasons mentioned in § 1.

The distributions of the variability brightness temperatures and the corresponding Doppler factors for each class of AGN are shown in Figures 1 and 2. It is interesting to note that all quasars and BL Lac objects are Doppler boosted ($T_{b,\mathrm{var}} > 10^{11}$ K). Very high values dominate the Doppler factor histogram for HPQs, as expected, and the peak of the distribution is $D_{\mathrm{var}} \approx 11$. The peak is lower for LPQs, around $D_{\mathrm{var}} \approx 8$. For BLOs, the Doppler factor distribution strongly concentrates around the lower values, peaking at $D_{\mathrm{var}} \approx 5$. The distribution is fairly smooth, suggesting that high values are not preferred, contradicting the belief that BLOs are rapidly variable sources. Radio galaxies, in turn, dominate the lower end of the distribution. The median and lower and upper quartile values for both $T_{b,\mathrm{var}}$ and D_{var} for each class of AGN are presented in Table 2. Looking at the median values of D_{var}, the HPQs are

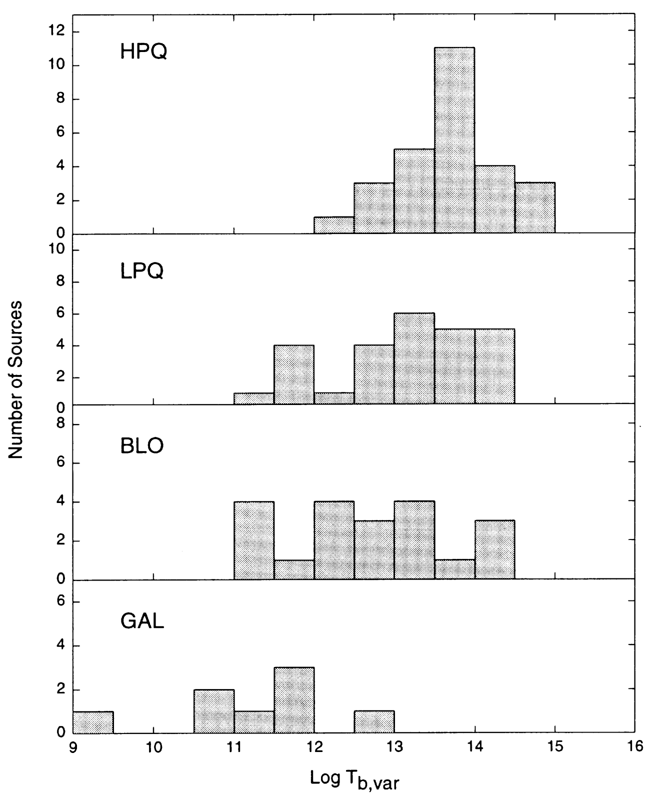

FIG. 1.—Distribution of $T_{b,\mathrm{var}}$

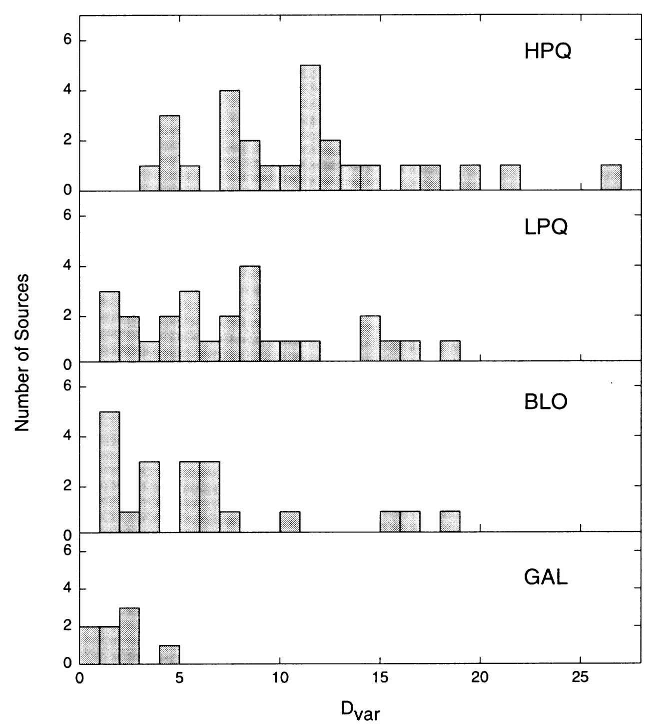

FIG. 2.—Distribution of D_{var}

TABLE 2

MEDIAN VALUES OF log $T_{b,\mathrm{var}}$ AND D_{var}

		log $T_{b,\mathrm{var}}$			D_{var}		
TYPE	N	Median	Lower Quartile	Upper Quartile	Median	Lower Quartile	Upper Quartile
HPQ......	27	13.84	13.29	14.06	11.13	7.29	13.18
LPQ......	26	13.34	12.72	13.71	7.58	4.72	10.10
BLO......	20	12.88	11.81	13.23	5.32	2.38	6.97
GAL......	8	11.55	10.78	11.87	1.96	1.07	2.45
ALL......	81	13.25	12.20	13.87	7.09	3.17	11.38

clearly the most strongly Doppler boosted objects, and the radio galaxies are the least boosted objects. There is a considerable difference between the Doppler factors of HPQs and LPQs, but the difference between LPQs and BLOs is much smaller.

To find out whether two (or more) classes of AGNs are drawn from the same $T_{b,\mathrm{var}}$ or D_{var} distribution, we performed the Kruskal-Wallis one-way analysis of variance on the observed variability brightness temperatures (or Doppler factors). The probabilities are shown in Table 3. It is evident that all quasars and BLOs differ from radio galaxies. The HPQs are not likely to be from the same distribution as either LPQs or BLOs. There is a fairly strong similarity between LPQs and BLOs, suggesting that the sources in these two classes may share the same parent population, at least partially, or that the Doppler boosting distributions of the parent populations are similar.

3.2. *A Comparison between D_{var}, D_{SSC}, and D_{eq}*

G&D state that the distributions of D_{SSC} and D_{eq} are similar and that the mean ratio $D_{\mathrm{eq}}/D_{\mathrm{SSC}}$ is on the order of unity. However, this does not establish the correctness of D_{SSC} or D_{eq}, since the two values are strongly correlated if the same radio data are used. This, and the effect of the time dependence of VLBI observations, are best noted if we write D_{SSC} and D_{eq} in terms of the observed VLBI brightness temperature, $T_{b,\mathrm{VLBI}} \propto S/\theta^2\nu^2$. For $\alpha = -0.75$ (with $S \propto \nu^{\alpha}$),

$$D_{\mathrm{SSC}} \propto T_{b,\mathrm{VLBI}}\,\theta^{0.37}\nu^{0.68}S_{\mathrm{X}}^{-0.18}\nu_{\mathrm{X}}^{-0.14} . \quad (5)$$

For D_{eq}, we have the relation

$$D_{\mathrm{eq}} \propto T_{b,\mathrm{VLBI}}\,S^{0.10}\theta^{-0.34}\nu^{-0.31} . \quad (6)$$

The values of D depend almost totally on the observed VLBI brightness temperature, which is a highly variable quantity from one observation to another because of the strong temporal variations in both the structure and the flux density of compact radio sources. Through $T_{b,\mathrm{VLBI}}$, the D values are also critically dependent on the VLBI observing frequency (cf. § 1). In addition, the two estimates of D are not independent, but rather $D_{\mathrm{SSC}} \sim D_{\mathrm{eq}}$ as a result of a similar dependence on $T_{b,\mathrm{VLBI}}$.

TABLE 3

KRUSKAL-WALLIS ANALYSIS OF VARIANCE

Type	P
HPQ/LPQ/BLO/GAL......	0.0000
HPQ/LPQ	0.0342
HPQ/BLO	0.0009
HPQ/GAL	0.0000
LPQ/BLO...................	0.1235
LPQ/GAL	0.0007
BLO/GAL	0.0060

We can test the quality of our variability Doppler factors only in an indirect way, since the true values of D for any AGN are unknown. When calculating D_{eq} (or D_{SSC}), any available sample of VLBI observations (S, θ, ν) is in principle equally suitable, since there are no turnover frequency measurements, the values change with time, and the X-ray data are not simultaneous in the case of D_{SSC}. We can therefore derive the equipartition Doppler factors by using different VLBI samples, and compare them with each other or with D_{var}. If D_{eq} (or D_{SSC}) is a good estimate of the true amount of Doppler boosting present in a source, the values derived from different data sets should be comparable. For this purpose, we calculated D_{eq} for a large and uniform sample of VLBI measurements at 22 GHz, namely the VSOP Pre-Launch Survey (PLS; Moellenbrock et al. 1996), and compared the results with those of G&D and our own D_{var}. Figure 3*a* shows a comparison between D_{eq}(PLS) and D_{eq}(G&D). The points are scattered, and it is obvious that they do not correlate very well. The plot of D_{var} and D_{eq}(G&D) is shown in Figure 3*b*. The correlation is slightly better, but it is undoubtedly best for the last pair, D_{var} and D_{eq}(PLS), shown in Figure 3*c*. The two independent "traditional" Doppler factors are less correlated with each other than with the variability Doppler factor. Using the Spearman rank correlation, we confirm that D_{eq}(PLS) and D_{eq}(G&D) hardly correlate at all. For the 41 sources in common, we get $P_S = 0.090$. A much better correlation is found between the variability Doppler factors and the two equipartition boosting factors. For D_{var} and D_{eq}(G&D), the correlation is $P_S = 0.002$ (48 sources), and for D_{var} and D_{eq}(PLS) $P_S = 0.0002$ (60 sources). The corresponding medians of the difference, calculated as $|\log D_1 - \log D_2|$, are 0.44, 0.31, and 0.28, respectively. (These results for D_{eq} generally apply for D_{SSC} as well.) The median difference between the variability and the two equipartition factors is $\sim\sqrt{2}$ times smaller than between the two independent equipartition factors. A natural conclusion is that essentially all scatter in the plots is caused by the errors in D_{eq} or, at any rate, that the errors in D_{eq} must be significantly larger than in D_{var}. This leads us to conclude that D_{var} is a much better estimate of the true Doppler factor.

The discussion in § 1 makes it clear why the PLS values of D_{eq} appear to be better than the G&D values. A major

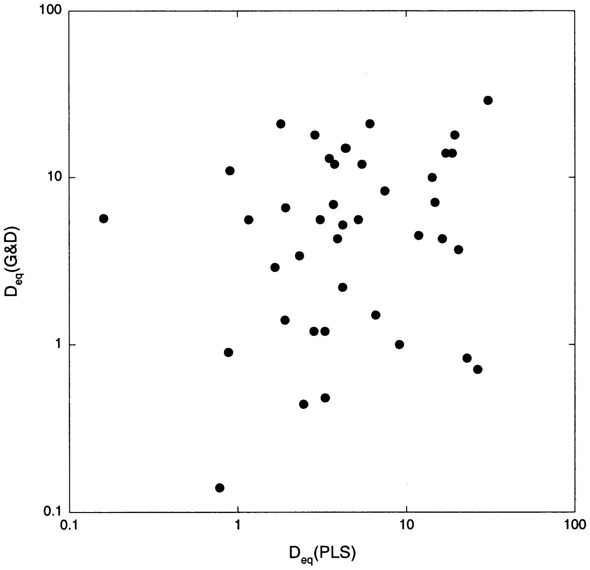

FIG. 3*a*

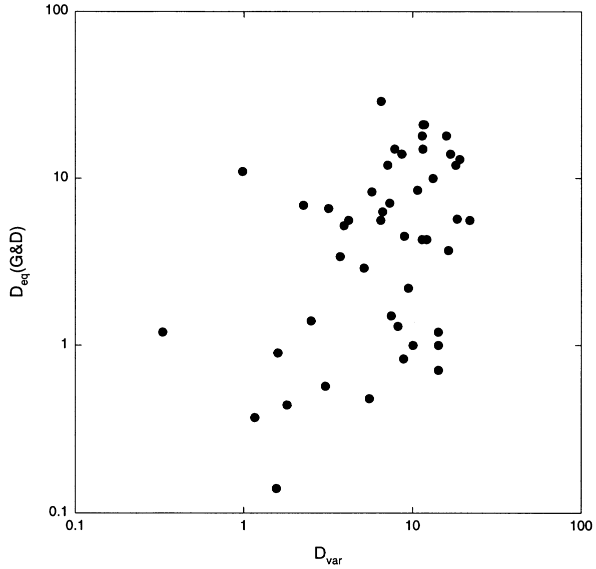

FIG. 3*b*

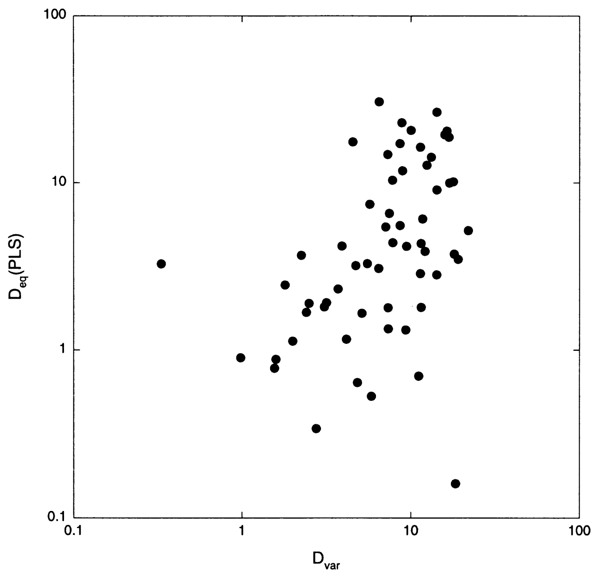

FIG. 3*c*

FIG. 3.—Comparison of Doppler factors calculated from three samples: (*a*) $D_{\rm eq}$(PLS) vs. $D_{\rm eq}$(G&D), (*b*) $D_{\rm var}$ vs. $D_{\rm eq}$(G&D), and (*c*) $D_{\rm var}$ vs. $D_{\rm eq}$(PLS).

fraction of the G&D VLBI data are taken at frequencies of 5 GHz or even lower, and relate to old components with brightness temperatures below the maximum $T_{b,\rm int}$.

Looking at the different Doppler factors, we note that the distribution of $D_{\rm var}$ is generally very similar to the distribution of $D_{\rm SSC}$ (or $D_{\rm eq}$) in the G&D sample. The median values of $D_{\rm var}$ are slightly larger for all classes of AGNs. The median for HPQs is $D_{\rm var} \sim 11$, compared to $D_{\rm SSC} \sim 9$; for LPQs it is ~ 8 compared to ~ 5, for BLOs ~ 5 compared to ~ 4, and for GALs ~ 2 compared to ~ 1, respectively. Some of the variation here may be accounted for by the different sample selections. In addition, the low-frequency VLBI data used in G&D generally result in an underestimation of $D_{\rm eq}$ (or $D_{\rm SSC}$). The same trends are also present in the comparison of the 48 common sources, shown in Table 4. Even if the medians of all classes are in approximate agreement, this is not true for individual objects. For example, for LPQs the difference between $D_{\rm var}$ and $D_{\rm SSC}$ for a source is ≥ 5 (and up to 15) for 75% of the sources. For HPQs the percentage is 60%, and for BLOs it is 35%. It is obviously possible to use the median values of $D_{\rm SSC}$ (or $D_{\rm eq}$) as guidelines in estimating the average Doppler boosting in each class of AGNs,

TABLE 4

MEDIAN VALUES OF $D_{\rm var}$ AND $D_{\rm SSC}$ FOR THE 48 OVERLAPPING SOURCES

Type	N	$D_{\rm var}$	$D_{\rm SSC}$
HPQ......	18	11.38	9.05
LPQ	11	8.16	4.60
BLO	14	5.32	3.50
GAL	5	1.16	1.20
ALL	48	7.98	4.75

but one should be cautious when using the individual source values in any calculations. For that purpose we suggest using the variability Doppler factor instead.

4. THE LORENTZ FACTORS AND VIEWING ANGLES

The Lorentz factors and viewing angles calculated with D_{var} from equations (3) and (4) are shown in Table 1 and Figure 4. Medians for each class of AGNs are presented in Table 5. Most of the HPQs are concentrated in the area of small θ_{var} and rather large Γ_{var}. The difference between the HPQs and the LPQs is evident, especially in the Lorentz factors. In contrast, the LPQs and BLOs have overlapping distributions with similar Γ_{var} but quite different θ_{var}. However, there is one BLO with a Lorentz factor of ~ 57. The apparent expansion speed, β_{app}, for 0219+42 is the largest in the sample, almost 15 (Marchenko et al. 1998). Most of the sources in our sample have Lorentz factors of ≤ 10. The true distribution of Γ could be slightly wider, but still, values such as that of 0219+42 stand out suspiciously. A value that large might be due to an exceptionally powerful jet, at the extreme end of the Γ distribution, but it is more likely that the VLBI expansion speed has been overestimated. The variability Doppler boosting factor for 0219+42 is well determined from several flares, so an underestimation of this quantity cannot be the reason for the large Γ value. If we calculate the median Γ_{var} for BLOs excluding 0219+42, we get 4.7. This actually makes the BLOs a quite separate group from LPQs. The Lorentz factors for GALs are modest, and the viewing angles are very large compared to the other classes of AGNs.

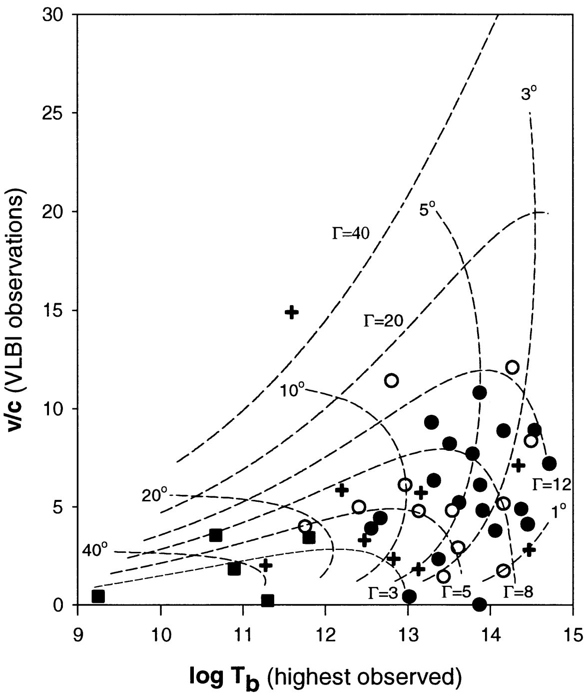

FIG. 4.—Lorentz factors Γ_{var} and viewing angles θ_{var}, with their dependence on the observable quantities β_{app} and $T_{b,var}$. *Filled circles*, HPQs; *open circles*, LPQs; *crosses*, BLOs; *filled squares*, GALs.

TABLE 5

MEDIAN VALUES OF Γ_{var} AND θ_{var}

Type	N	Γ_{var}	θ_{var}
HPQ......	19	7.40	4.06
LPQ......	12	5.99	4.54
BLO......	9	5.84	8.61
	8[a]	4.70	8.61
GAL......	5	2.44	29.63
ALL......	46	6.47	5.17

[a] BLOs excluding 0219+42.

We have also compared our variability Lorentz factors and viewing angles with those calculated from D_{SSC} and D_{eq} in the Guerra & Daly (1997) sample. Again, the general trends agree, except for LPQs. The median values of Γ_{var} and Γ_{SSC} (or Γ_{eq}) for HPQs are approximately 7 and 9, respectively. For LPQs, we get a median of 6, while Guerra & Daly get a value slightly higher than that for HPQs. For BLOs, the estimates are rather similar, around 5, and they also agree well for GALs. Looking at the individual estimates tells a different story. The range of Lorentz factors for both the SSC and the equipartition cases is very large, with values extending up to near 90. Even the general distribution without the most extreme values is rather large, from ~ 5 to over 30. The variability estimates occupy a uniform group with a rather tight peak between 2 and 12 (quasars and BLOs). Without 0219+42, the maximum Γ_{var} is 15.5.

The median values for the viewing angles θ_{var} and θ_{SSC} (or θ_{eq}) are both approximately 4° for HPQs, 5° and 6° for LPQs, 9° and 14° for BLOs, and 30° and $\sim 100°$ for GALs, respectively. The ranges of the viewing angles of all estimates agree quite well (usually $\leq 20°$), except for GALs, for which we find a much narrower distribution, with a maximum value of 50°. The explanation for the rather small viewing angles and comparatively large flow speeds is that the few radio galaxies in our sample of compact flat-spectrum sources are not representative of the whole population, being more like borderline cases between radio galaxies and quasars. We also compared the 19 sources in common between our sample and the Guerra & Daly (1997) sample. The results are shown in Table 6. The previous remarks apply here as well. The problematic BL Lac object 0219+42 is not included here, so the variability Lorentz factor for BLOs is notably lower than for LPQs.

We have generally used the same β_{app} data as Guerra & Daly (1997), collected by Vermeulen & Cohen (1994), with a few new values, so that the differences in Γ and θ are due to different estimates of D. The use of inaccurate Doppler factors is clearly reflected in the large spreads of Γ_{SSC} and θ_{SSC}. The better quality of our variability Doppler factor is very well demonstrated in the realistic distributions of Γ_{var} and θ_{var}.

TABLE 6

MEDIAN VALUES OF Γ AND θ FOR THE 19 OVERLAPPING SOURCES

Type	N	Γ_{eq}	θ_{eq}	Γ_{SSC}	θ_{SSC}	Γ_{var}	θ_{var}
HPQ......	5	8	6	8	6	9.9	3.6
LPQ......	7	8.9	9	8.6	10	5.3	8.5
BLO......	4	4.15	7.5	4.25	11.5	3.5	6.65
GAL......	3	1.15	60	1.15	56	2	29.6
ALL......	19	6	7	5	10	5.7	5.1

In the basic unified models, the viewing angle is the decisive parameter in the classification of AGNs (e.g., Barthel 1989). There is a torus of obscuring material around the core region of the AGN, and the axis of the relativistic jet is perpendicular to this disk. At large angles, the observer sees the AGN and its obscured core as a radio galaxy. Then, depending on the type of radio galaxy (FR I or FR II), as the viewing angle decreases, the observer sees first a low-polarization quasar and then a high-polarization quasar (FR II) or a BL Lac object (FR I). This scenario holds nicely for our variability viewing angles. However, if all classes of AGNs originate from the same parent population, the Lorentz factors should be about the same for all of them. Looking at the values of $\Gamma_{\rm var}$, it is quite clear that all classes form separate groups of their own (BLOs excluding 0219+42), ordered by increasing $\theta_{\rm var}$ and decreasing $\Gamma_{\rm var}$. The BLOs and GALs are clearly different from quasars. Nevertheless, the difference in $\theta_{\rm var}$ for HPQs and LPQs is negligible, suggesting that intrinsic differences, instead of the viewing angle, are decisive for the appearance (e.g., optical polarization) of these objects. It may also be that the combination of Γ and θ produces either a HPQ or a LPQ, in contrast to the basic unified models. Our sample also excludes a number of steep-spectrum LPQs. All the BLOs in our sample are radio selected, and their range is between 0° and $\sim 30^\circ$. The X-ray–selected BL Lac objects should have larger viewing angles than radio-selected BL Lac objects (Padovani & Urry 1992), but there is no room left in the $\theta_{\rm var}$ range exclusively for them. This indicates that the intrinsic differences are more important than the orientation for BLOs. However, in view of the small number of sources in each class, these conclusions must be treated with caution.

5. CONCLUSION

The variability Doppler boosting factors calculated in this paper have been shown to be as precise and reliable as currently possible, and hence they should be useful in determining the Lorentz factors and viewing angles for individual sources. This opens up a wealth of possibilities in AGN research. With the aid of individual Lorentz factors and viewing angles, we can start modeling the source. For example, we can remove the influence of relativistic boosting on true intrinsic properties, such as the luminosity or the peak frequency. It will also be possible to hunt down misclassified and unusual objects (such as microlensed sources) by looking for deviant source parameters. It is also possible that the strength of the outflow Γ correlates with the amount of microvariability or with the gamma-ray emission. The connection between the radio properties (e.g., the outflow speed) and the gamma-ray properties of AGNs will be investigated in Lähteenmäki & Valtaoja (1999b, Paper IV). Preliminary results have already been presented in Lähteenmäki et al. (1997) and Lähteenmäki & Valtaoja 1999a). More effort is needed in order to produce better and larger samples of Doppler factors and other source parameters. More accurate VLBI measurements with careful estimations of the expansion speed are essential. It would also be a good idea to obtain simultaneous VLBI and TFD monitoring data, since the parameters needed (D, Γ, θ) depend on time. We even lack the redshifts for a few sources.

The most important part in the determination of the individual source parameters is played by the method by which the Doppler factor is estimated. Doppler factors determined by the traditional method from VLBI and X-ray data, and also equipartition Doppler factors, since they are closely related, are rather useless in this sense. Even if they can be used in statistical studies giving the parameters for various classes of AGNs, they will not give accurate values for individual sources and thus should not be used for the modeling. Of course, the determination of the variability Doppler factor is not without problems. The problems involved in the modeling of the variability data are presented in Paper I. Some of the sources are probably still misclassified. For example, 1308+32 is a source with both quasar and BL Lac–like properties. We have chosen to call it a quasar, while many others regard it as a BLO. The quality of VLBI expansion speed data is another concern. Any $\beta_{\rm app}$ exceeding ~ 10 should be treated with caution. The extreme case of the BL Lac object 0219+42 is a good example of a $\beta_{\rm app}$ most likely overestimated. This is obviously a serious problem, since in a small sample a single source with an incorrect expansion speed can change the outlook of a whole class of AGNs. One should be careful in drawing conclusions from flux-limited samples, since these tend to pick the most extreme sources of each class, thus bringing forward selection effects. Compensation for selection effects is to some extent possible, but the number of sources we have in each class at present is too small for any very strong conclusions.

A typical radio-loud quasar has Doppler and Lorentz factors of ~ 10 and a viewing angle of $< 5^\circ$. A typical BL Lac object has Doppler and Lorentz factors of ~ 5 and a viewing angle of $< 10^\circ$. The quasars clearly differ from BL Lac objects. The radio galaxies in our sample typically have Doppler and Lorentz factors of ~ 2 and viewing angles of $< 50^\circ$. We find clearly different Doppler factors for HPQs and LPQs (11.1 versus 7.6). However, the difference between the two groups of quasars is not very clear when looking at the Lorentz factors or the viewing angles alone. It seems that a combination of a more rapid flow speed and a smaller viewing angle—not merely the viewing angle, as in basic unification—produces a HPQ or a LPQ.

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