Higher Order Time Series analysis of AGN light curves

Dimitrios Emmanoulopoulos

8th ENIGMA meeting, 6-8 September 2006, Espoo Finland

Landessternwarte Heidelberg

ENIGMA

Overview

- What we mean with "Higher Order Time Series analysis (HOTSA)"
- The dynamical system of Mkn 421
- TS and dimensionality
- Method of Surrogates
- Conclusions

HOTSA

Linear Gaussian Systems

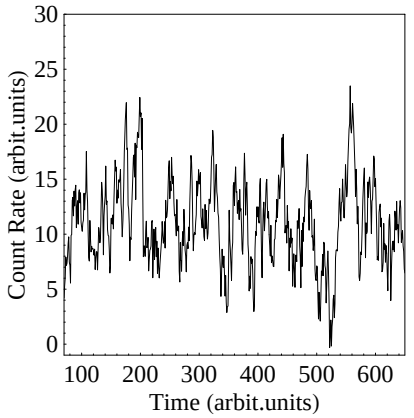
$$\left.\begin{aligned}\mathcal{L}x_1(t) &= y_1(t)\\ \mathcal{L}x_2(t) &= y_2(t)\end{aligned}\right\} \Rightarrow \mathcal{L}\left[\lambda_1 x_1(t) + \lambda_2 x_2(t)\right] =$$

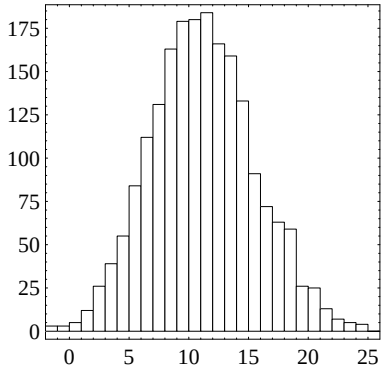
$$\lambda_1 \mathcal{L}x_1(t) + \lambda_2 \mathcal{L}x_2(t) =$$

$$\lambda_1 y_1(t) + \lambda_2 y_2(t)$$

$$\boxed{x_t = Ax_{t-1} + Be_t + C}$$

HOTSA





HOTSA

1^{st} and 2^{nd} statistical moments are sufficient to characterize the system.

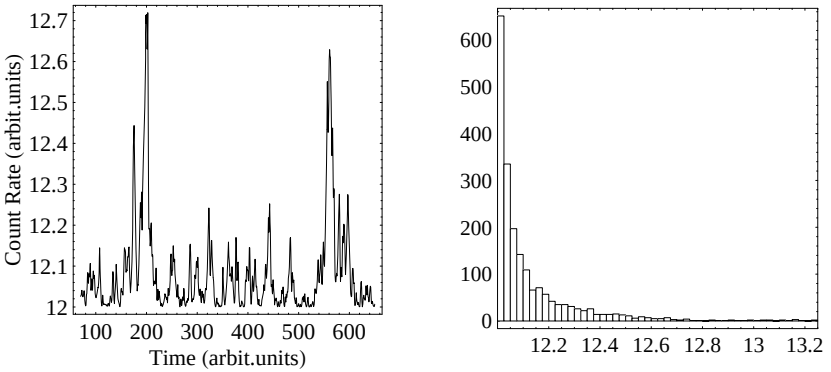
SF: $S_x(\tau) = \frac{1}{N(\tau)} \sum [x(t+\tau) - x(t)]^2$

ACF: $ACF(\tau) = \frac{E[(x(t)-\mu)(x(t+\tau)-\mu)]}{\sigma^2}$

HOTSA

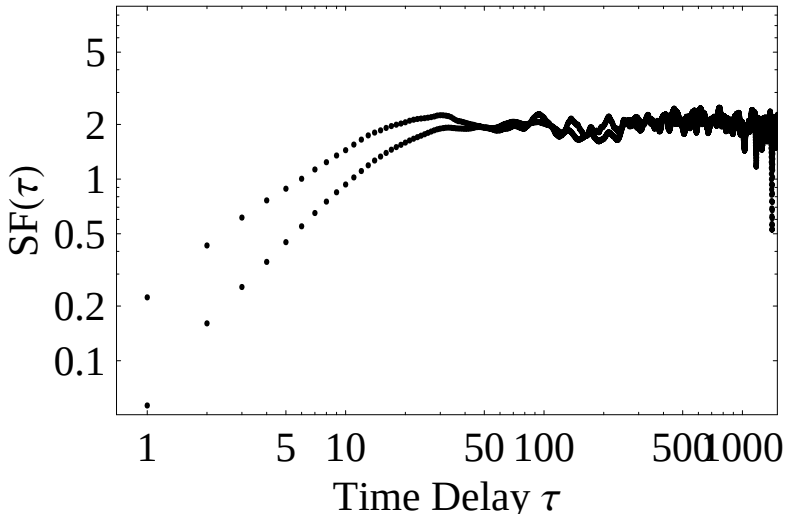
Non-Linear Systems

$$x_t = Ax_{t-1}^{1/3} + Bx_{t-1}^{1/2} + e_t + C$$



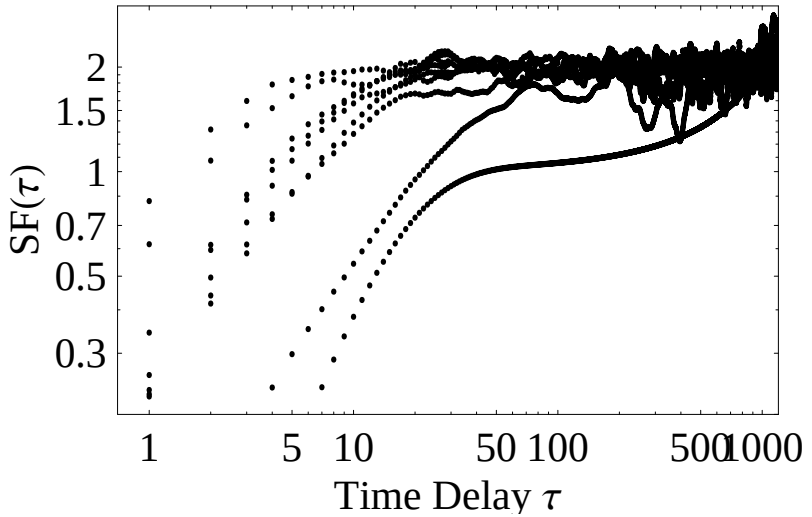
HOTSA

The SFs



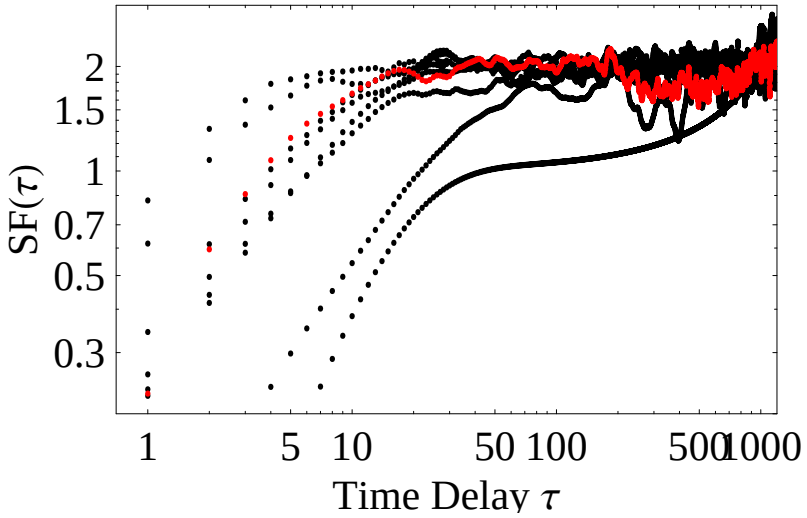
HOTSA

Which one is which?



HOTSA

Which one is which?



HOTSA

AGN light curves exhibit non-linear behavior.

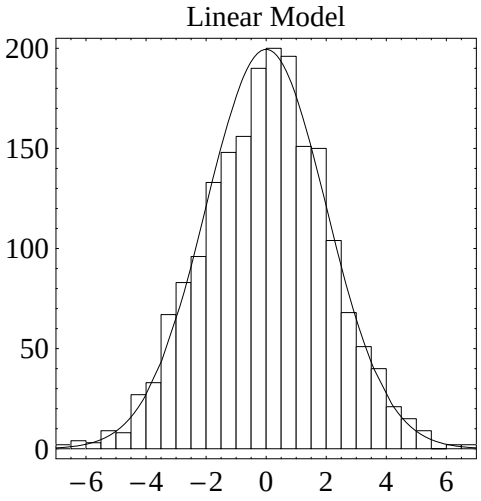
(Leighly et al. 1997, Green et al. 1999)

Nonlinear tests

- The distribution of increments $\Delta x(t) = x(t+1) - x(t) \longrightarrow$ "Intermittency" (e.g. Subba Rao, Priestley Lessi, 1997, Guégan, 1994).
- Specification of the correlation dimension $C_m(r)$ (Grassberger, Phys.Rev.Lett. 1983,Theiler, Phys. Rev. A 1987) $\longrightarrow$ "Method of surrogates" (Theiler et al., Physica D 1992)

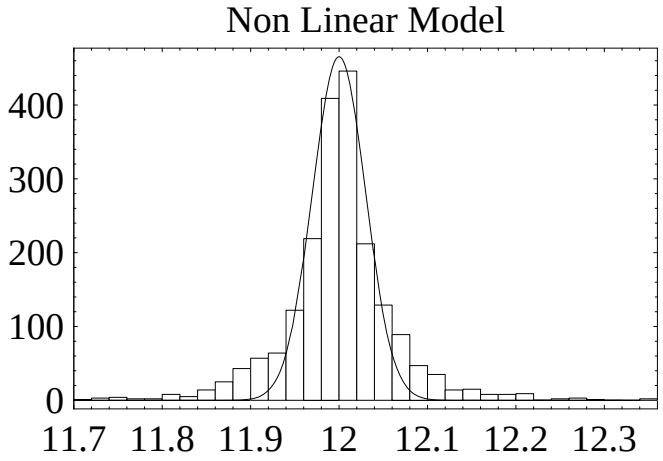
HOTSA

Distribution of increments $\Delta x(t) = x(t+1) - x(t)$



HOTSA

Distribution of increments $\Delta x(t) = x(t+1) - x(t)$



HOTSA

Distribution of increments $\Delta x(t) = x(t+1) - x(t)$

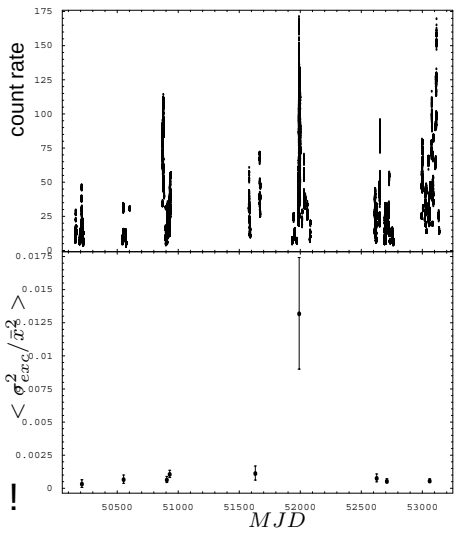
Exponential tails $\longrightarrow$ Larger probability to have "Burst events".

With Gaussian linear statistics

- We loose valuable information included in the data.
- We conclude to false/fake time scales.
- We do not consider the (dynamical) noise component which is inherent in the source (e.g. Lawrence et al. 1987, McHardy 1987,Vio et al. 2005)

The dynamical system of Mkn 421

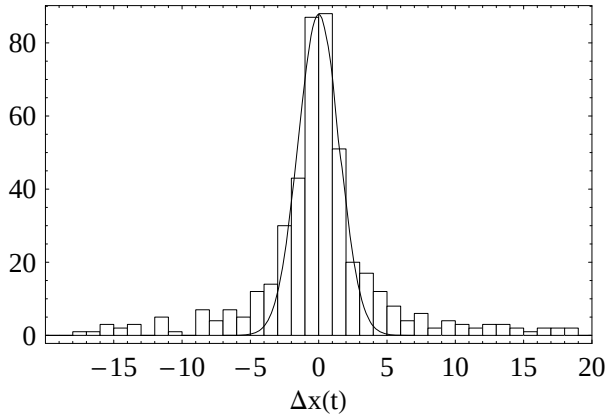
For Mrk 421



Non Stationarity !

The dynamical system of Mkn 421

For Mrk 421



Existence of intermittent events!

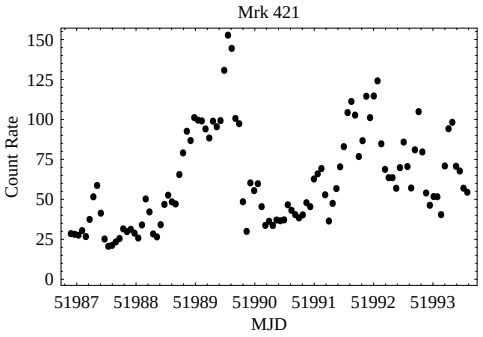
The dynamical system of Mkn 421

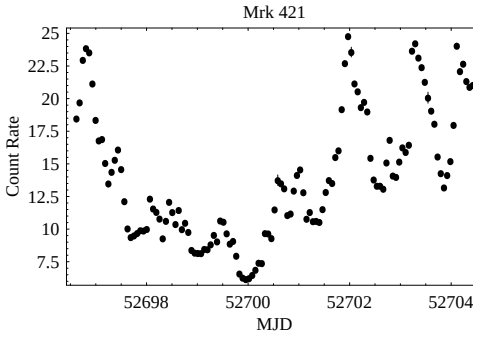
The long term LC is the outcome of a nonlinear dynamical physical system.

The dynamical system of Mkn 421

The long term LC is the outcome of a nonlinear dynamical physical system.

What about shorter periods?Is there any trace of nonlinearity?



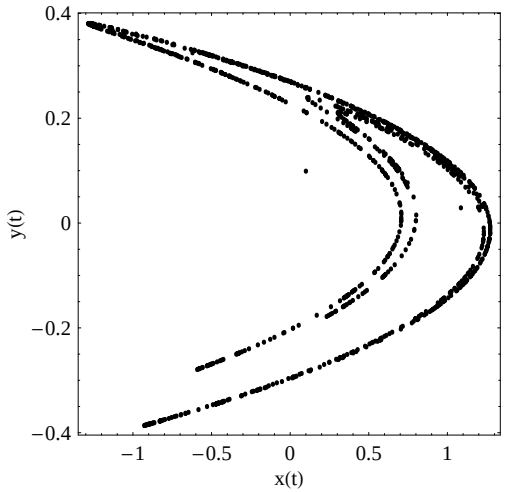


Dimensions in TS

- Capacity dimension (fractal dimension) D_0
- Information dimension D_1
- Correlation dimension D_2
- Pointwise dimension $D_{\mathrm{p},j}$
- Generalised dimension D_{q}
- Lyapunov dimension D_{L}

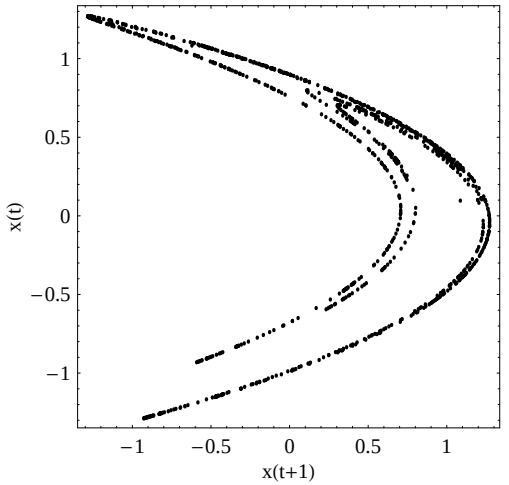
Dimensions in TS

Henon Map



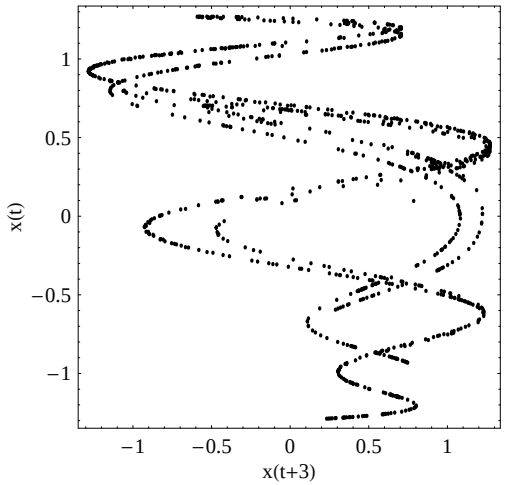
Dimensions in TS

Henon Map with delayed variables (2D, τ=1)



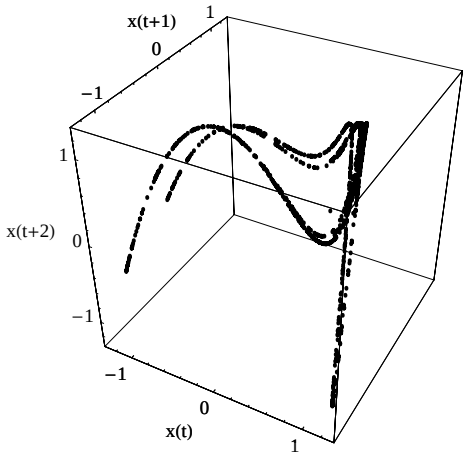
Dimensions in TS

Henon Map with delayed variables (2D, τ=3)



Dimensions in TS

Henon Map with delayed variables (3D, τ=1)



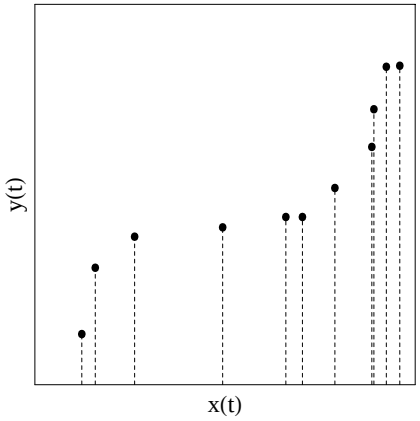
Dimensions in TS

We form hyperspheres around the points and we check for the existence of other points inside them.
The mean probability is

$$C_m(r) = \frac{\sum_{i=1}^{T}\sum_{j=1,i\neq j}^{T} H(r - ||\vec{x_i} - \vec{x_j}||)}{(T-1)T}$$

with
$\vec{x_i} = (x_i, x_{i+\tau}, \cdots, x_{i+(m-1)\tau})$ and $T = N - (m-1)\tau$

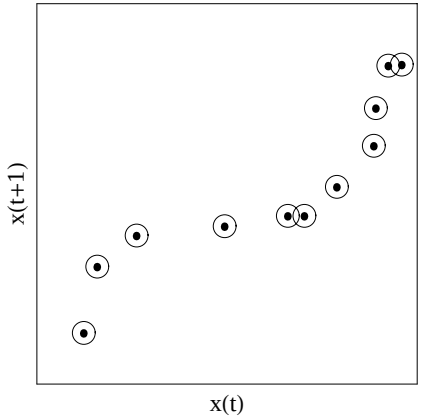
Dimensions in TS



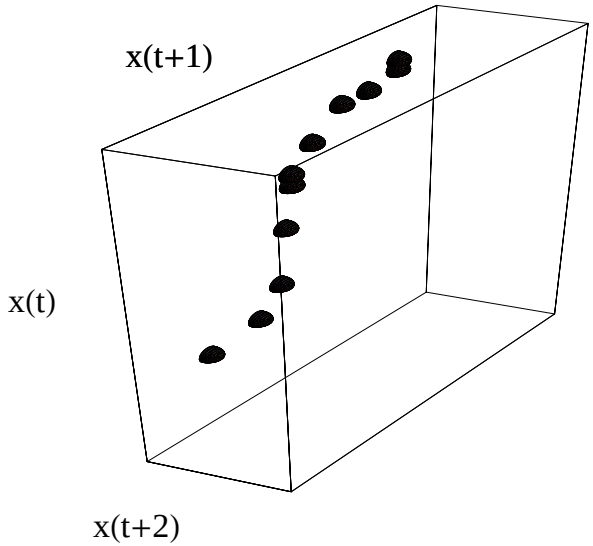
Dimensions in TS

$x(t)$

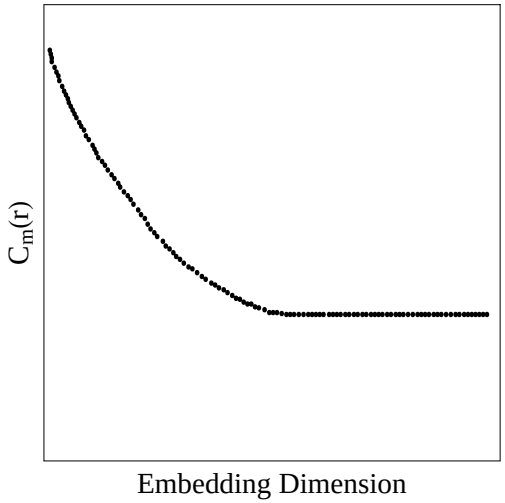
Dimensions in TS



Dimensions in TS

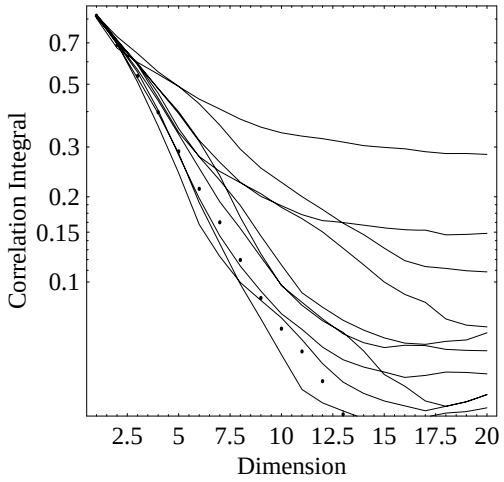


Dimensions in TS



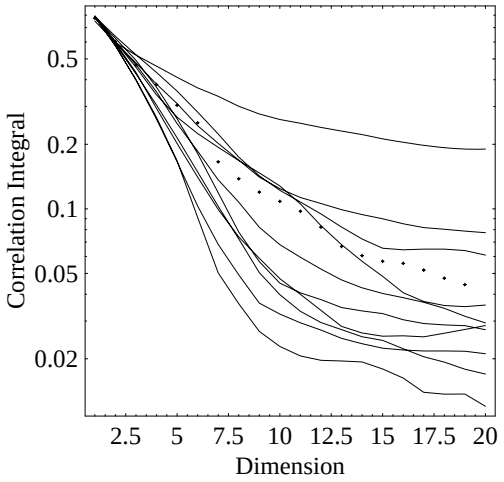
Method of surrogates

For Mrk 421 the Method of surrogates:



Method of surrogates

For Mrk 421 the Method of surrogates:



Method of surrogates

- More data !!!
- The light curves have a lot of dynamical noise.

Non linearity in small time scales can not be ruled out.

Conclusions

- Higher statistical orders are necessary for the full description of the system.

Conclusions

- Higher statistical orders are necessary for the full description of the system.
- Measurements errors and data gaps should always be included in every analysis method.

Conclusions

- Higher statistical orders are necessary for the full description of the system.
- Measurements errors and data gaps should always be included in every analysis method.
- Extended statistical studies of large data sets can reveal possible dynamical states of the source.

Conclusions

- Higher statistical orders are necessary for the full description of the system.
- Measurements errors and data gaps should always be included in every analysis method.
- Extended statistical studies of large data sets can reveal possible dynamical states of the source.
- The main purpose of the TS analysis is the prediction.

Last Conclusion

Thanks a lot!!!